5E Lesson Plan - Volume of Cones (12.4)

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Title of Lesson: Volume of Cones
Date of Lesson: 30 November 2012
Length of Lesson: 50 Minutes
Name/Level of Course: 6th grade, IB Pre-algebra

Why Is This Lesson Appropriate for Middle School Students?:
This lesson is middle school appropriate because it allows the student to have a hands on activity, appealing to their sensory needs, and also allows them the opportunity to work hands-on with their peers and share ideas.

Technology Lesson?: Yes, ActivExpression

Source of the Lesson:

- Adapted from The National Council of Teachers of Mathematics, Illuminations, "Popcorn, Anyone?". Retrieved from http://illuminations.nctm.org/LessonDetail.aspx?id=L797.
- Online Blog: "Runde's Room", 2012. Retrieved from http://www.rundesroom.com/search?q=volume.
- Cindy Neuschwander, "Sir Cumference and the Sword in the Cone: A Math Adventure" 2003.
- Mcgraw-Hill, Pre-Algebra, Section 12.4

Concepts:

Volume is the the quantitative measurement of the capacity of a three-dimensional object. Understanding the concept of volume is important for everyday life because volume is everywhere around us. It is what fills the soda bottles, how much ice cream in a waffle cone, and the amount of pumpkin in a can to make pumpkin pie. It can be defined by formulas for simple shapes, such as prisms, cones, and cylinders. The calculations involve cubic units of measurement, for example cubic centimeters. The ratio between a cylinder and a cone of the same base and height is 1:3. Formula for the volume of a cone is $V = \pi r^2 h$ & formula for the volume of a cylinder is $V = \pi r^2 h$.

Objectives:

1.) Students will be able to describe the formula for the volume of a cone.
2.) Students will be able to apply the formula for the volume of a cone.
3.) Students will be able to solve for missing components of the formula given a volume.

State Standards:

Benchmark Number:	MA.6.A.3.4

Benchmark Description:	Solve problems given a formula.
Subject Area:	NGSSS: Mathematics
Grade Level:	6
Body of Knowledge:	Algebra
Big Idea:	BIG IDEA 3 - Write, interpret, and use mathematical expressions and equations.

Benchmark Number:	MA.6.G.4.3
Benchmark Description:	Determine a missing dimension of a plane figure or prism given its area or volume and some of the dimensions, or determine the area or volume given the dimensions.
Subject Area:	NGSSS: Mathematics
Grade Level:	6
Body of Knowledge:	Geometry
Supporting Idea:	Geometry and Measurement - Geometry and Measurement

FACT Used:

KWL Variations (#27), pg 116-119 & Look Back (#29) pg 121-123 from "Mathematics Formative Assessment" by Keeley and Tobey -- combined into one chart

Safety Concerns:

Do not put beans in mouth.

Materials List and Advanced Preparations:

1 foam rectangular prism, cylinder and cone
7 hollow cones (1 for each group)
7 hollow cylinders (1 for each group)
**cones & cylinders must have the same height & base for each group
Dry beans (enough to fill the 7 hollow cylinders)
Copies of "KWLH Handout" for each student
Copies of the Post-Assessment for each student
21 ActivExpression Clickers (for Promethean Board-Mentor teacher has them)
Copies of "Working with Cones" worksheet for each student
Copies of "Challenge Problems" worksheet for each student
Copies of Interactive Notebook "Volume of Cones" for each student

ENGAGEMENT **Time: 10 Minutes**		
What the Teacher Will Do	**Probing/Eliciting Questions**	**Student Responses and Misconceptions**
Introduce cones by reading parts of "Sir Cumference and the Sword in the Cone."		
Display a rectangular prism, cylinder & cone.	Who can tell me what we did the first time we were here? Does anyone remember that formula? What about last time we were here, what did we do? What was the formula for that? What's different about the characteristics of a cone from the last 2 shapes (cylinder & rectangular prism)?	Found the volume of a rectangular prism. V=length*width*height Found the volume of a cylinder. V=area of the base*height or V=pi*radius squared*height It only has one base. It has one vertex. It has a circular base like the cylinder.
Distribute the KWLH worksheet to each student. (**FACT**) Instruct the students to take a few minutes to fill out the FIRST TWO columns.		

Transition Statement: Today you guys are going to be mathematicians and write a general formula for finding the volume of a cone.

EXPLORATION **Time: 20 Minutes**		
What the Teacher Will Do	**Probing/Eliciting**	**Student Responses and**

	Questions	Misconceptions
Have the students in groups of 3 for activity. (Desks will be prearranged for when they enter the room) Remind the students of the formula for the volume of a cylinder. (Have it written on board)	"Who remembers the formula for the volume of a cylinder?"	$V = pi * r^2 * h$
	"How did you discover that formula?"	By layering coins in a cylinder. Area of the base times height.
	"Do you think the formula for the cone is similar to the formula for the cylinders? Why?"	Yes, they both have a circular base.
	"How do you think we can determine the formula for the volume of a cone?"	Answers may vary; check for misconceptions. By "cutting off some" of the cylinder.
	"Why can't we use centimeter cubes or coins?"	The cone comes to a point and is not square. There will be empty space at the bottom where the coins won't fit.
Recall the formula for area. $A = \pi r^2$ Also, recall 'keeping your answer in terms of pi': Say: Instead of multiplying your answer by 3.14, which is an approximation of pi, just leave your answer in terms of pi, without multiplying. For example, If your answer is going to be 12 x 3.14, instead just write 12π.		

Tell the students that we have given them all the materials needed to discover the formula for the volume of a cone. Have them work within their group come up with the formula. Say: Today you get to be a mathematician and come up with the formula for finding the volume of a cylinder. We have given you everything you need in order to do this. We will be walking around if you have any questions. Remember to keep all of the beans out of your mouths and on the desks only, you may pour the beans from one to another or anything else you think it appropriate to find the volume of the cone. Instruct students to write the formula on the bottom of their KWLH handout when they have came up with a formula. Call on a group we know has the right formula. Write the formula the student/group says on the board, and ask for a quick poll on how many people agree with this formula, using the ActivExpression clickers. Distribute an ActivExpression clicker to each student and instruct the students to type in their name into the clicker. Then the teacher will also pass	For guidance- "Why might the formula be similar to the formula for cylinder?" "Do you think a cone is a *part* of a cylinder?" "How might you find this ratio between the volume of a cylinder and a cone?" "How could you find how the volume of the cone compares to the volume of the cylinder?" "How much stuff will it take to fill up a cylinder compared to the amount it takes to fill up a cone?" "So who would like to tell the class what you got as the formula for the volume of a cone?"	The bases are the same size. Yes because it looks like the cone will fit inside of the cylinder. Fill one and pour the beans into the other one. $V = \frac{1}{3} \pi * r^2 * h$

out the worksheet "Working with Cones" which will also be displayed through the ActivExpression program. The teacher will pass out the challenge problems to any groups that finish early. The questions on the worksheet will be the ActiveExpression Questions. The ActivExpressions allows the teacher to display the question on the Promethean board and the students type in their choice into their handheld. The students enter their name and it shows on the board when each student has entered in their answer.		

Transition Statement: So I heard a lot of discussion on how you can find the volume of a cone, so let's talk about what we all have discovered.

EXPLANATION **Time: 8 Minutes**		
What the Teacher Will Do	**Probing/Eliciting Questions**	**Student Responses and Misconceptions**
Instruct the students to place the cones, cylinders, and beans in the tray or bin in the middle of the desks.		
Ask if any group got something different and hear their group's explanation, and correct any misconceptions that occured to get them to the wrong formula. Demonstrate that 1 cone filled up about ⅓ of the cylinder's space.	We all figured out that the formula is $V = ⅓ \text{ pi} * r^2 * h$, can one group explain to me how they got this formula?	It took the beans from 3 cones to fill up the cylinder OR 1 cone filled up about ⅓ of the cylinder's space.
Give the students a few minutes		

to fill out the THIRD and FOURTH columns of the KWLH handout.		
The teacher will display the questions from "Working with Cones" and the results from the exploration. The teacher will review the questions and correct any misconceptions.	"How did you get B as your answer?" "Explain to me why C works in this case" For Questions 3 & 4 of the worksheet: "Who remembers how we solved for a missing dimension, ie length, width & height, when we knew that volume of a rectangular prism?" "And how did we do so with finding the height or radius with a given volume of a cylinder?" "So how do we find the height of the cone when we are given volume of a cone and radius?" (Work out Question 3) "How do we find the radius the cone with given volume and height?"	Plugged it into the equation then solved for missing variable Divided the volume by the height & width. Plugged it into the equation then solved for missing variable Divided the volume by the area of the base Divided the volume by the height Solve the equation for the formula for height and solve. Divide the volume by 3 times the area of the base Solve the equation for radius and solve. Divide the volume by height times 3 times radius squared.

Transition Statement: Let's talk about some of the instances in where would use the volume of a cone and where we see cones in our everyday lives.

ELABORATION **Time: 4 Minutes**		
What the Teacher Will Do	**Probing/Eliciting Questions**	**Student Responses and Misconceptions**
Interactive notebook!!! -- This time includes students' examples of where they see cones and why the formula is useful.	When dealing with cones it is important to know where exactly the height of the cone is. On your notebook	The length from the center of the base to the vertex of the cone.

Consider a real-world application to include in the interactive notebook.	page, there is a picture of a cone. Can someone please describe to me how we would measure the height of the cone? What did you discover the formula for finding the volume of the cone to be? Where do we see cones? What are some examples of when we would find the volume of a cone?	V=1/3*pi*radius squared*height Safety cones on the street, waffle cones, snow cone cups, water cups at doctor offices, some coffee filters are cones, party hats How much ice cream, water, or coffee grounds fit into the cone

Transition Statement: As always, it's time for the quiz. Please work individually and not with a partner.

EVALUATION **Time: 7 Minutes**		
What the Teacher Will Do	**Probing/Eliciting Questions**	**Student Responses and Misconceptions**
Teacher will pass out the post tests and allow the students 5 minutes to complete it. After all the students have completed the post tests, the teacher will ask for volunteers to read the question and how they solved for the answer.		

Note for us:
Remember to use during lesson
**Wait time when teacher asks a harder question*
**Share with your partner*

K-W-L-H Handout

Name:_____

Instructions: Fill out the columns when appropriate.

What do you already **K**now about cones?	What do you **W**ant to find out about cones?	What have you **L**earned about cones?	**H**ow did you learn it?

The formula for the _____ of a _____ is

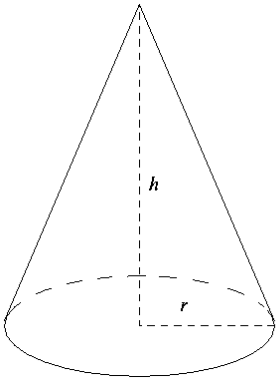
_____.

Working with Cones

Name:_____

Remember to show all work.

Find the volume of the cone.

1.) 

radius = r = 2 cm

height = h = 9 cm

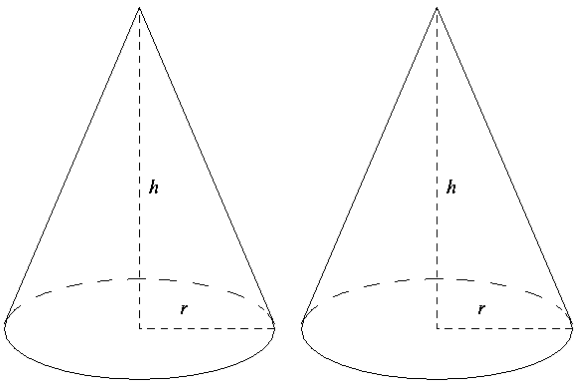
A) V = 18□cm^3

B) V = 12□cm^3

C) V = 54□cm^3

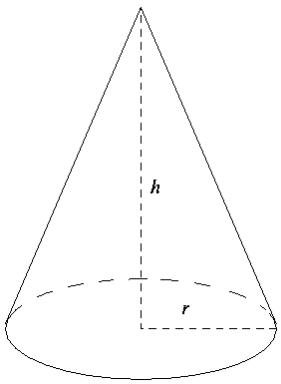
D) V = 30□cm^3

Find the volume of the cone.

2.) 

diameter = d = 10 in.

height = h = 3 in.

3.) 

Find the height of the cone.

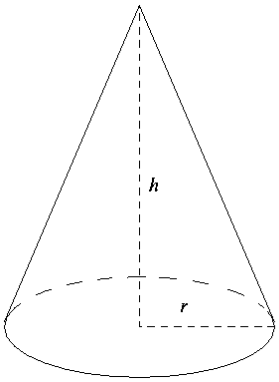
r = 3 cm

$V = 30\pi cm^3$

Find the radius of the cone.

4.) $h = 6$ ft.

$V = 8\pi ft^3$

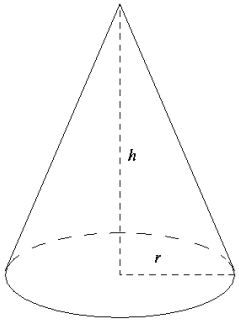


^^^Above pictures just used as a demonstration. Google docs doesn't have a cone as a "drawing". Actual worksheet will have corrected cones with different dimensions. Answers submitted to ActivExpression & student will have scrap paper.

Post-Assessment
Mrs. Chomat
30 November 2012 Name: _____

1.) Describe how to find the volume of a cone.

2.) Find the volume of the cone.

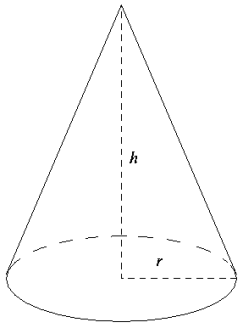


r = 4 cm
h = 6 cm

Show all work here:

A) 24π cm^3
B) 8π cm^3
C) 96π cm^3
D) 32π cm^3

3.) What is the height of a cone if the volume is 12π $in.^3$ and the radius is 2 in.?
Show all work.



h = _____ in.

Solutions & Rubric

Post-Assessment (5 points possible)

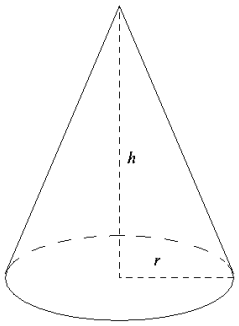
1.) Describe how to find the volume of a cone. (1 pt)

volume equals ⅓ times pi times the radius squared times height

OR volume equals ⅓ times the area of the base times height

OR written out in symbols

2.) Find the volume of the cone.



(1 pt)

r = 4 cm

h = 6 cm

Show all work here:

(Box here)

$V = (1/3) * pi * 4^2 * 6$

A) 24 cm^3

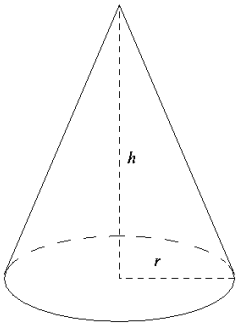
B) 8 cm^3

C) 96 cm^3

D) 32 cm^3 (1 pt)

3.) What is the height of a cone if the volume is 12 $in.^3$ and the radius is 2 in.?

Show all work.



$12pi = ⅓ * 2^2pi * h$

Solve for h: $(12pi)/(1/3*4pi) = 9$ (1 pt)

h = ___9__ in. (1 pt)

Extra Challenge Problems

Name: _____

1) Which has a greater volume?

a) A cylinder with a radius r=2ft and a height h=3ft, or a cone with a diameter d=6ft and a height h=3ft. **12*pi ft cubed(cylinder), 9*pi ft cubed (cone)**

b)A cylinder with a diameter d=1ft and a height h=2ft, or a cone with a radius r=1ft and a height h=2ft. **pi/2 ft cubed (cylinder), ⅔*pi ft cubed (cone)**

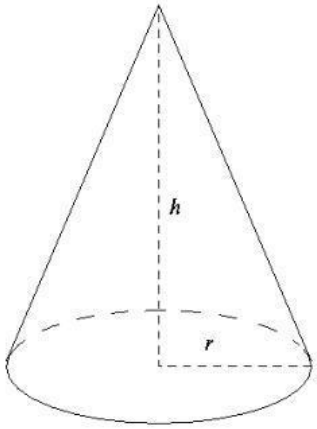
2) The volume of a cone is *27*cubic inches with a radius of 3 inches. The volume of a cylinder is *36*cubic inches with a diameter of 4 inches. True or false: the height of the cylinder and cone are different? **height of cone and cylinder are the same--h=9 inches, false**

3) A cylinder and a cone have the same radius of 4 cm, but different heights and volumes. If you are given that the volume of the cylinder is half the volume of the cone and that the height of the cone is 12 cm, what is the height of the cylinder?
Vcone=1/3*pi*r^2*h=(1/3)(16)(12)*pi=64*pi= 2*Vcylinder
Vcylinder=pi*r^2*h= Vcone/2= 64*pi/2=32*pi
Vcylinder=16*pi*h=32*pi
h=2 cm

Interactive Notebook Page:

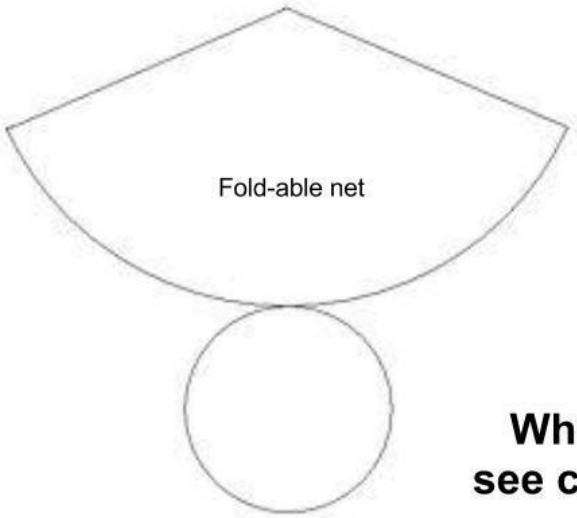
Name:_____

12.4 Volume of Cones



Height

Formula



Example

Where do we see cones in our lives?